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Experimental study and mathematical modeling of convective fish drying process

Summary

To study the equilibrium state of the fish and the air flow was carried out experimental studies of equilibrium moisture content of herring in the air stream with the specified parameters. The average air flow rate was 1.2 m·s^{-1} at a temperature of +30°C. Relative humidity was regulated from 85% to 30%. The article presents the curves of drying of the sabrefish is based on experimental data for different values of temperature and air humidity in the drying installation. The values of the current humidity fish was calculated based on the ratio of the mass of moisture in the product to the mass of absolutely dry matter. The differential equation of moisture transfer fish to the conditions of the experiments was solved by numerical method. The results of the calculations of the sabrefish convective drying in the period of falling speed for the two significantly different conditions are presented in the article. The experimental points agree satisfactorily with the results of calculations, with the exception of the beginning of the specified period.

Key words: *convective drying, mathematical modeling*

Modelowanie i badania eksperymentalne konwekcyjnego procesu suszenia ryb

Streszczenie

W celu zbadania stanu rozkładu wilgotności równowagowej na profilu głębokości tkanki ryby w zależności od parametrów powietrza suszącego, przeprowadzono badania eksperymentalne. Średnia prędkość przepływu powietrza wynosiła 1,2 m·s^{-1}. Wilgotność względna powietrza wynosiła 85% i 30%, a temperatura 20 i 30°C. Przedstawiono krzywe suszenia na podstawie wyznaczonych danych eksperymentalnych dla różnych wartości temperatury i wilgotności powietrza w instalacji suszącej. Wykonano również obliczenia na podstawie zaprezentowanych formuł matematycznych. Wyznaczone eksperymentalnie punkty są zgodne z wynikami obliczeń, z wyjątkiem początkowego okresu suszenia.

Słowa kluczowe: *suszenie konwekcyjne, modelowanie matematyczne*

Symbols:

A – coefficient of moisture transfer [m^2·h^{-1}],	a – Dimensionless coefficient of moisture transfer [-],
A_p – Equilibrium coefficient of moisture transfer [m^2·h^{-1}],	q – Dimensionless gradient magnitude of moisture transport
T – Temperature of the air [°C],	d – Moisture content of the air, [g·kg^{-1}]
U – Fish moisture content [-],	u – Dimensionless fish moisture content [-],
U_p – Equilibrium fish moisture content [-],	Bi – mass exchange of Bio
J – Intensity of surface moisture [m·h^{-1}],	α – coefficient of moisture exchange [m·h^{-1}],
t – Time [h],	τ – Dimensionless time [-],
X – Coordinate [m],	x – Dimensionless coordinate [-],
W – Fish humidity [%],	K – Empirical coefficient [m^2·h^{-1}].

Introduction

The drying process is one of the main stages in the preparation of smoked and dried fish products. Factors affecting the internal mass transfer in the fish and the external mass transfer from the fish to the drying agent, to a large extent affect the duration of the process and its energy consumption, and determine the organoleptic characteristics of the finished product and shelf life (Lykov, 1956; Risker, 1957; Voskresenskiy, 1966; Buckle, 1995; Olokor and Omojowo, 2009).

Calculations of thermal dryers are usually reduced to the determination of the heat consumption for drying in the parameter values of the drying agent, the recommended technological instructions. It does not take into account the patterns of moisture transport between the fish and the drying agent. Working medium in the drying chamber is a mixture of dry air and water vapor, the share of which in the mixture is characterized by the value of its partial pressure. In this case the partial vapor pressure in the boundary layer near a wet material is always greater than in the bulk air.

Under the action of the difference of the pressures is vapor diffusion from the material into the air. Under other equal conditions the rate of evaporation from the surface of the material depends on the partial pressure of vapor in the air or its moisture content. The process of drying fish was terminated when the boundary conditions when the moisture content of the material reaches equilibrium moisture content. The isotherms of desorption of moisture depend on the chemical composition of the material condition and installed only experimentally.

Description of the experimental setup and methodology of experiments

To study the equilibrium state of the fish and the air flow was carried out experimental studies of equilibrium moisture content of herring in the air stream with the specified parameters. Equilibrium moisture content a whole round herring were determined in a stream of air moving at an average speed of 1.2 m·s^{-1} at a temperature of +30°C. Relative humidity was regulated from 85% to 30%. Moisture content of fish was determined by the standard method.

Studies to determine the equilibrium moisture content of fish in the air flow for the optimization of drying process and cold smoking is conducted at an experimental installation, allowing to maintain over a long period of time constant air parameters: temperature, relative humidity and speed. The scheme of installation is represented in Fig. 1 (Syslov et al., 2007).

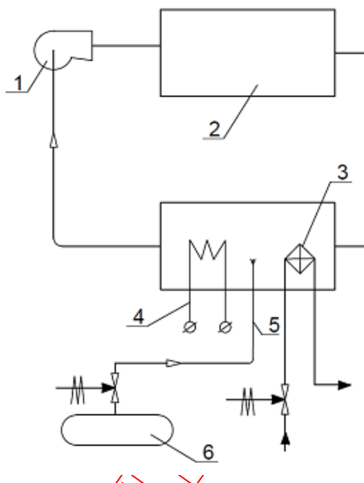

Fig. 1. The scheme of experimental installation: 1 – fan; 2 – drying chamber; 3 – cooler; 4 – electric heater; 5 – humidifier; 6 – steam generator

Rys. 1. Schemat instalacji eksperymentalnej: 1 – wentylator; 2 – komora suszenia; 3 – chłodnica; 4 – grzejnik elektryczny; 5 – nawilżacz; 6 – wytwornica pary

The results of experimental research

In Fig. 2 present the curves of drying sabrefish is based on experimental data for different values of temperature and air humidity in the drying installation (fish humidity, *W* [%, kg moisture per kg dry matter]). The values of the current humidity W fish were calculated based on the ratio of the mass of moisture in the product to the mass of absolutely dry matter. The influence of the air temperature in the drying installation can be seen from a comparison of curves 1 and 4, 2 and 3: at 20°C the process of drying fish is characterized by low intensity, due to insufficient capacity of thermoliposomes.

The increase in air temperature to 30°C intensificare process, but also leads to deterioration of the drying conditions (lines 3 and 4) of the studied objects, as the dryness of the surface layers leads to the formation of a "crust" that prevents thermoplastique. Reducing the moisture content of the air in the drying installation (from 10.8 to 7.4 g moisture per kg dry air) also leads to the intensification of the drying process (lines 2 and 1, 3 and 4), so the values of the influence factors of temperature and humidity should be considered rational for the process. The initial moisture content of fish in experiment 1 was higher than in the experiment 2. However, high intensity of mass transfer in experiment 1 led to the fact that the final moisture content of fish in experiment 1 is even slightly lower than in the experiment 2 (see Fig. 2).

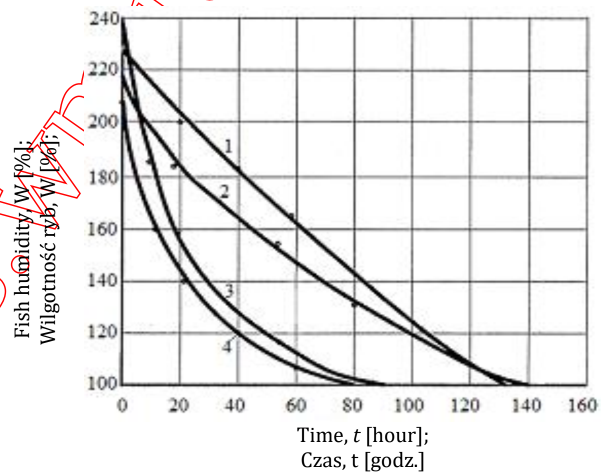

Fig. 2. The dependence of the drying sabrefish experimental results on the temperature (T) and the moisture content (d) of the air: 1 – T = 20°C; d = 7.4 g; 2 – T = 20°C; d = 10.8 g; 3 – T = 30°C; d = 10.8 g; 4 – T = 30°C; d = 7.4 g.

Rys. 2. Spadek wilgotności (W) w rybie gatunku karpiowatych, w czasie (t) suszenia, dla różnych temperatur (T) i zawartości wilgoci (d) powietrza suszącego: 1 – T = 20°C; d = 7,4 g; 2 – T = 20°C; d = 10,8 g; 3 – T = 30°C; d = 10,8 g; 4 – T = 30°C; d = 7,4 g.

Description of the experimental setup and methodology of experiments

The differential equation of moisture transfer fish to the conditions of the experiments can be written in the following form (Lykov, 1956; Lykov and Mikhailov, 1963):

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial X}\left(A \cdot \frac{\partial U}{\partial X}\right) \qquad (1)$$

where: t is the time, X – coordinate, $U = U(X,t)$ is the moisture content, A is coefficient of moisture transfer.

Boundary conditions for differential equation (1):

$$U(X,0) = f(X),$$

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$$\left(\frac{\partial U}{\partial X}\right)_{X=0}=0,$$
$$A\cdot\left(\frac{\partial U}{\partial X}\right)_{X=L}=-J \tag{2}$$

where J is the intensity of surface moisture.

In the period of falling speed of solving problems on finding the moisture field was proposed a formula for moisture exchange between the body surface and the environment (Lykov, 1956):

$$J=\alpha\cdot\left(U(L,t)-U_p\right) \tag{3}$$

Where: α is the coefficient of moisture exchange related to the difference in moisture contents; U_p – the equilibrium moisture content.

Formula (3) applicable to the period of constant drying rate, for J = const, the coefficient α will increase continuously with decreasing moisture content because the moisture content on the body surface $U(L,t)$ decreases during the drying process.

The differential equation of moisture transfer (1) with boundary conditions (2) under the assumption that mass exchange features A and α do not change, was analytically solved by Lykov (1956). But the coefficient of moisture transfer fish A depends on moisture content, it is not to be removed from under the sign of the derivative in equation (1). The empirical formula (Ginsburg et al., 1980) was used:

$$A(U)=A_p+K\cdot\left(U-U_p\right) \tag{4}$$

Dimensionless quantities were introduced:

$$u=\frac{U-U_p}{U_0-U_p},$$
$$x=\frac{X}{L},$$
$$\tau=\frac{t\cdot A_p}{L^2},$$
$$a=\frac{A}{A_p}=1+k\cdot u$$
$$k=K\cdot\frac{U_0-U_p}{A_p} \tag{5}$$

From equations (5) express quantities and substitute in (1) and (2). After the conversion, will receive the differential equation of moisture transfer and boundary conditions in dimensionless form (Naumov et al., 2015):

$$\frac{\partial u}{\partial\tau}=\frac{\partial}{\partial x}\left(a\cdot\frac{\partial u}{\partial x}\right) \tag{6}$$

$$u(x,0)=1,$$
$$\left(\frac{\partial u}{\partial x}\right)_{x=0}=0,$$

$$a\cdot\left(\frac{\partial u}{\partial x}\right)_{x=1}=-Bi\cdot u(1,\tau) \tag{7}$$

Where the mass exchange of Bio $Bi=\alpha\cdot L/A_p$.

The boundary value problem (6)-(7) cannot be solved analytically, since the mass transfer coefficient is a function of moisture content. For the numerical solution of the problem in Mathcad, we introduce the dimensionless gradient magnitude of moisture transport q and equation (6) to (7) can be written as:

$$\frac{\partial u}{\partial\tau}=\frac{\partial q}{\partial x},\quad q=a\cdot\frac{\partial u}{\partial x}, \tag{8}$$

$$u(x,0)=1,$$
$$q(0,\tau)=0,$$
$$q(1,\tau)=-Bi\cdot u(1,\tau) \tag{9}$$

To solve the boundary problem it is necessary to find the parameters of the particular sample of fish (Naumov at al., 2015). In Fig. 3 presents the experimentally obtained dependence of the equilibrium moisture content of herring at 30°C of the moisture content of the supply air.

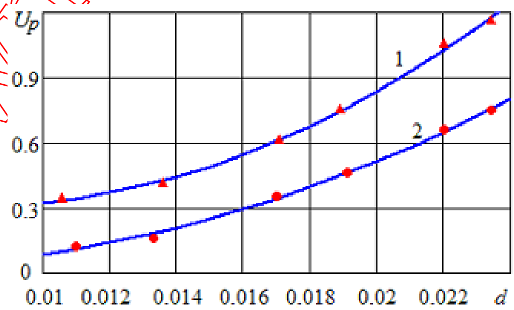

Fig. 3. Dependence of the equilibrium moisture content of herring (kg of moisture per kg of dry matter) from the moisture content of the air (g moisture per g of dry air) at T = 30°C: 1 – whole fish, 2 – pieces. Points – experimental data; curves: calculation formulas (10)-(11)

Rys. 3. Zależność wilgotności równowagowej dla śledzia (U_p - kg wilgoci na kg suchej masy) od wilgotności powietrza (d – g wilgoci na g suchego powietrza) w T = 30°C: 1 – całe ryby, 2 – kawałki. Punkty – dane eksperymentalne; krzywe: formuły obliczeniowe (10)-(11)

The method of least squares in Mathcad was established that the lowest variance is obtained by approximation based on a second-order polynomial. For whole fish:

$$U_{p1}(d)=0.531-56.69\cdot d+3623\cdot d^2 \tag{10}$$

For pieces:

$$U_{p2}(d)=0.068-18.65\cdot d+2058\cdot d^2 \tag{11}$$

The deviation of the calculation results by formulas (10)-(11) from the experimental data does not exceed 3%.

Show you how to assess the value of the coefficient of moisture transfer α from the experimental data (Syslov et al, 2007; Naumov at al., 2016). In the period of constant drying rate:

$$\frac{\partial U}{\partial t}=Q=const \tag{12}$$

Will printeriem both sides of equation (1) on the thickness of the layer L:

$$\int_0^L Q\, d\, X = \frac{1}{L}\cdot\int_0^L \frac{\partial}{\partial X}\left(A\cdot\frac{\partial U}{\partial X}\right) d\, X,$$

$$Q\cdot L = \left(A\cdot\frac{\partial U}{\partial X}\right)_{X=L} \qquad \textbf{(13)}$$

(13) subject to boundary conditions (2)-(3) can be obtained

$$\alpha(t) = \frac{Q\cdot L}{U(L,t)-U_p} \qquad \textbf{(14)}$$

Value *U(L,t)* decreases, and since *Q = const*, from (14) follows the well-known fact (Lykov, 1956): *α(t)* increases the period of constant drying rate.

Based on the accepted model in the period of falling drying rate *α = const*. Then this constant value can be estimated at the boundary point between these periods of drying according to the formula:

$$\alpha \approx \frac{\frac{\partial W}{\partial t}\cdot L}{W - W_p} \qquad \textbf{(15)}$$

Where: *W* is the average moisture content of a sample of fish defined in the experiments (Syslov et al, 2007; Naumov et al., 2016).

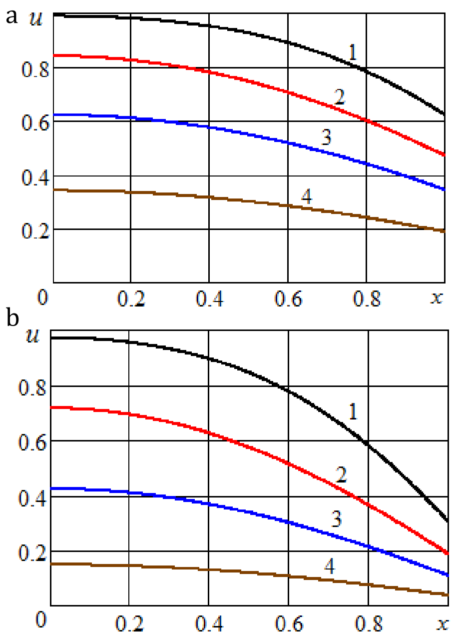

Fig. 4. Profiles of the dimensionless moisture content sabrefish at different points in time: 1) τ = 10; 2) τ = 30; 3) τ = 60; 4) τ = 120; a) T=20°C, d =10.8 g moisture per kg dry air; b) T=30°C, d =7.4 g moisture per kg dry air

Rys. 4. Profile wilgotności równowagowej (u) dla ryb gatunku karpiowatych w różnych punktach głębokości ryby (x), po czasie suszenia: 1) t = 10; 2) t = 30; 3) t = 60; 4) t = 120; przy: a) T = 20°C, d = 10,8 g wilgoci na kg suchego powietrza; b) T = 30°C, d = 7,4 g wilgoci na kg suchego powietrza

In Fig. 4-6 presents the results of calculations of the convective drying of sabrefish in the period of falling speed for the two significantly different conditions *T* = 20°C; *d* =10.8 g moisture per kg dry air and *T* = 30°C; *d* =7.4 g moisture per kg dry air. Fig. 5 shows the evolution of the profiles of dimensionless moisture content with time. Fig. 4 and 5, one can estimate the increase of drying speed sabrefish with the increase of temperature and decrease of moisture content of the air supplied.

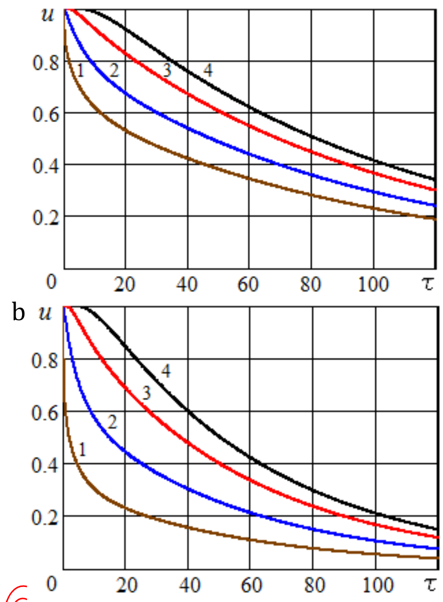

Fig. 5. Change in time of dimensionless moisture content sabrefish: 1) on the sample surface (x = 1); 2) x = 0.8; 3) x = 0.5; 4) x = 0.0; a) T=20°C, d = 10.8 g moisture per kg dry air; b) T=30°C, d =7.4 g moisture per kg dry air

Rys. 5. Zmiana wilgotności równowagowej (u) dla ryb gatunku karpiowatych od czasu (τ): 1) x = 1(na powierzchni próbki); 2) x = 0,8; 3) x = 0,5; 4) x = 0.0 (w centrum próbki); dla: a) T = 20°C, d = 10,8 g wilgotnoci na kg suchego powietrza; b) T = 30°C, d = 7,4 g wilgotnoci na kg suchego powietrza

$$\bar{u}\,(t) = \int_0^1 u(x,\tau)\; d\; x \qquad \textbf{(16)}$$

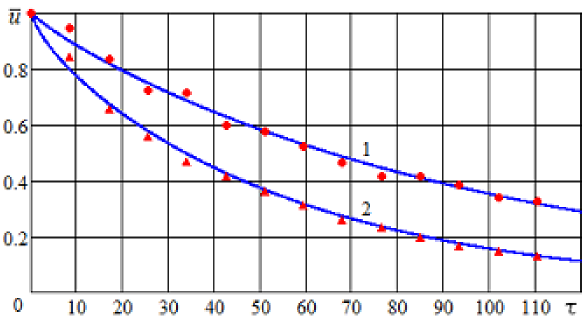

Fig. 6. Curves of dehydration during drying sabrefish in the period of falling speed (dimensionless variables): 1) T = 20°C, d = 10.8 g moisture per kg dry air; 2) T=30°C, d =7.4 g moisture per kg dry air. Points – experimental data, lines – result of calculation

Rys. 6. Krzywe dehydratacji ($\bar{u}$) w czasie (t) suszenia dla ryb gatunku karpiowatych: 1) T = 20°C, d = 10,8 g wilgotności na kg suchego powietrza; 2) T = 30°C, d = 7,4 g wilgotności na kg suchego powietrza. Punkty - dane eksperymentalne, linie - wyniki obliczeń

In Fig. 6 average dimensionless moisture content, calculated on the thickness of the sample (16) (Naumov et al., 2015) compared with the experimental data on drying sabrefish in the period of falling speed. The experimental points agree satisfactorily with the results of calculations, with the exception of the beginning of the specified period. Perhaps the beginning of the period falling speed was not accurately determined during the experiments. But during

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the period of constant drying rate of the considered model, as mentioned above, it is not fair.

Conclusions

The obtained experimental data of the equilibrium moisture content intact is a whole herring, which are approximated by a polynomial of 2nd order.

Differential equation of moisture transfer and boundary conditions expressed in dimensionless form.

The numerical solution of the problem in Mathcad, taking into account the experimental dependence of the equilibrium moisture content of the fish.

The results of calculations of mass-transfer Bio number from the dimensionless parameters defining the boundary conditions in the real range of their change.

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